Zeros & Factoring

Seeing More Than Procedures

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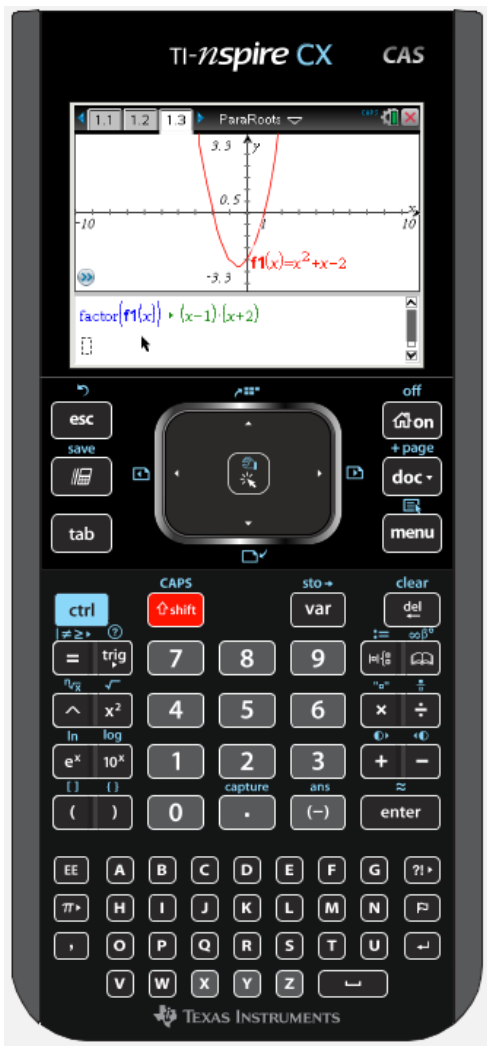

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Zeros & Factoring: Seeing More Than Procedures

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https://www.dropbox.com/s/aatsz3yimal7rj4/Factors%20%26%20Zeros%20-%20SD%20480p.mov?dl=0

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Transition Among Representations

- Shifts between concept image and concept definition (Tall & Vinner, 1981).
  - Each have an affect on the other as symbolic meaning is negotiated.
- Process of building symbolic meaning is mediated by interactions among two worlds (the “real” world of *physical reality or ideas* and the world of the *symbols that represent these ideas*. (Kaput, Blanton, & Moreno, 2008; Lapp, Ermete, Brackett, & Powell, 2013; Lapp & Cyrus, 2000)

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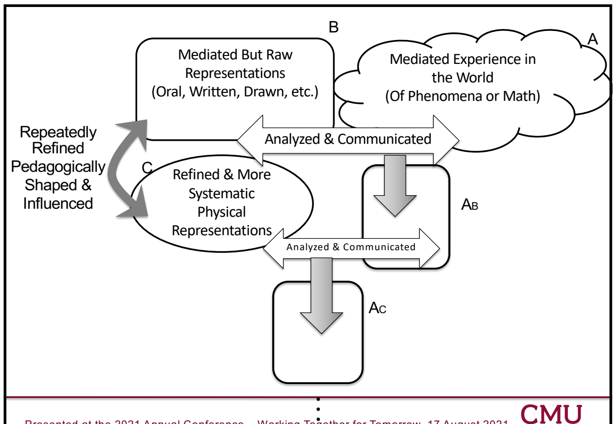

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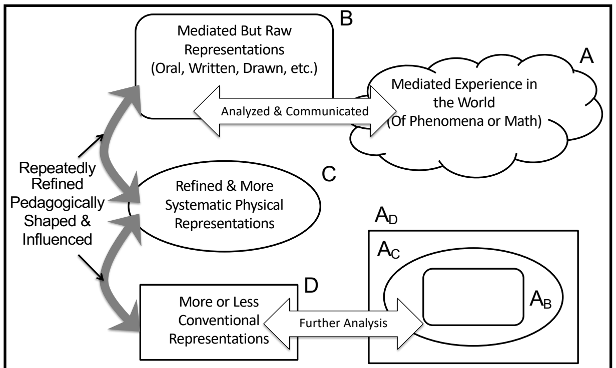

Continuing the process of symbolization Kaput, Blanton, & Moreno (2008)

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What Makes Active Learning “Active”?

- Students share control of the learning process.
  - Make decisions about what to do
  - Real World applications (not just word problems)
  - Debate the reasonableness of ideas
  - Justification of claims/observations

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What Makes Active Learning “Active”?

- Teachers act as diagnosticians and not as the sole conveyers of knowledge.
  - Teacher as reflective listener
  - Teacher as orchestrator

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What Facilitates an Active Classroom?

Thought provoking questions requiring more than a simple response.

- Opening Discussion Questions
- Labs/Activities for deep and detailed exploration

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What Facilitates an Active Classroom?

- Physical layout of the room
  - Teacher not in front
  - Students in tables for discussion
  - Student Grouping

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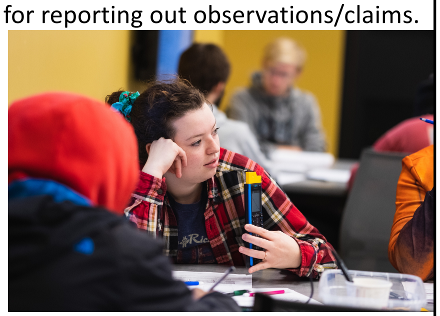
What Facilitates an Active Classroom?

Opportunity for reporting out observations/claims.

- Within working groups (safe environment to share)
- Whole class (after group has discussed)



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Home Run Derby (Authentic Problems to Motivate)

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Home Run Derby

Suppose we want to know how far the softball was hit and predict how far from home plate the ball landed given only this small portion of the video. Discuss in your groups the following questions.

- What questions might we need to ask?
- How might we collect data to answer these questions?

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Home Run Derby

Discuss in your groups the following questions.

- What are some possible mathematical models we could use to describe the path of the ball?
- Based on data we have collected, what are some ways we could find a specific model to fit our data?
- Once we have a model, how would we determine when it hit the ground?

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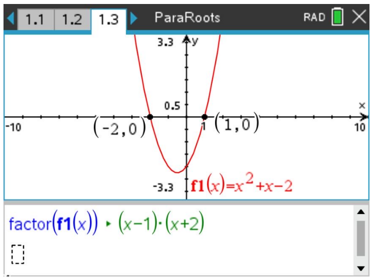
Transition Among Representations

- Once we have motivated the need for a process for finding zeros, we need to help students connect various representations in order for them to see the power of factoring in this process. (Lapp, Ermete, Brackett, & Powell, 2013; Lapp & Cyrus, 2000)



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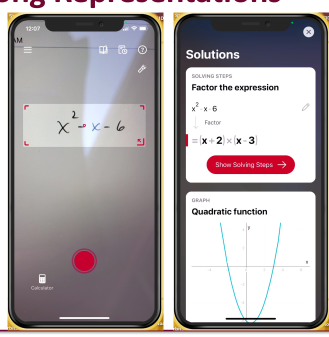
Transition Among Representations

- The same exploration we can do with the TI-Nspire CX CAS can also be accomplished with free technology such as Photomath or Wolfram Alpha (although not as dynamically).



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Parabolas and Roots: Graphs & CAS

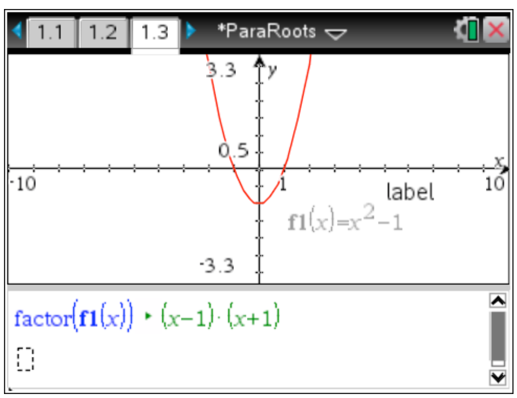
Materials

- *TI-Nspire™ CAS handheld*

Introduction to the Investigation

In this activity, you will use both algebraic and graphical representations simultaneously to solve quadratic equations and make observations about the graphs and symbolic form of the equations. In addition, you will make use of the Computer Algebra System (CAS) built into the TI-Nspire™ CAS handheld to look for patterns across the representations. One way the CAS is useful is that it enables you to see more than one representation of mathematical ideas on the same page allowing you to see how changes in one representation influence the changes in another representation.

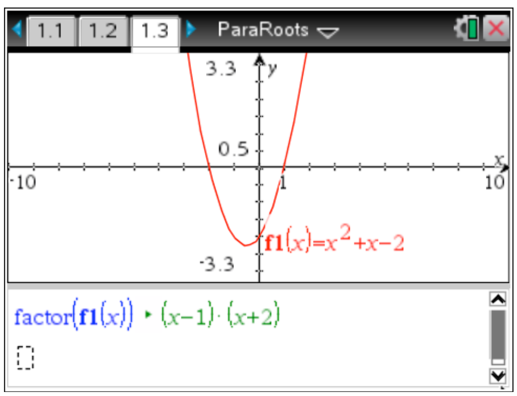
For example, we could have a graphical and symbolic representation of a parabola on the same page. Take the function $f(x) = x^2 - 1$ pictured below. Here we can see both the graph and the factored form of the function's algebraic expression on the same page.



Steps

Consider the function, $f(x) = x^2 + x - 2$.

1. Open the *ParaRoots.tns* document.
   - This file has a split screen with a **Notes** page on the bottom with the factored form of the equation and a **Graphs** page on the top.
2. First, label the *x*-intercepts of the equation so that you can see them as you explore the activity.
   - To do this, press menu and select the **Points & Lines** submenu and choose **3:Intersection Point(s)**. Click on the parabola and then the *x*-axis. This will place a point at the intersection of the parabola and the axis. To see the coordinates of the *x*-intercepts, choose **7:Coordinates and Equations** from the **Actions** submenu and click on each of the intersection points.



Explore:

You are free to change any of the representations on the **Graphs** pane of the page found at the top, whether it is the graph itself or the symbolic representation of the function. As you change one representation of the function, everything else will change to match the alterations you have made. You can change the graph by either editing the expression for the function or by pulling and manipulating the graph itself.

3. Take some time to explore the situation by manipulating the function, jotting down anything you discover or find interesting as you explore. Don't forget to include some rough sketches in the spaces provided on the next page. (If you need more room there are four more panes on the last page of this handout.

Parabolas and Roots: Graphs & CAS

✎ Sketch your screens below:

✎ Record your observations below:

Focused Questions:

Describe patterns you noticed while manipulating the graph.

Parabolas and Roots: Graphs & CAS

Describe parts of the graph and factored form of the function that appeared to be related no matter how you manipulated the graph.

When you pull the graph above the *x*-axis, describe what happens to the factorization.

Make a hypothesis about any connections you see between the factored form of the function and the features found on the graph.

For each of the *x*-intercepts shown on the graph, describe what you notice when the *x*-coordinate is plugged into the factored form of the function. Be specific here and explain what happens to each factor as well as the resulting product. Explain how your observations are related to the features on the graph.